

4. Two forces $\mathbf{F}_1 = (3\mathbf{j} + \mathbf{k})\text{ N}$ and $\mathbf{F}_2 = (4\mathbf{i} + \mathbf{j} - \mathbf{k})\text{ N}$ act on a rigid body. The force \mathbf{F}_1 acts at the point with position vector $(2\mathbf{i} - \mathbf{j} + 3\mathbf{k})\text{ m}$ and the force \mathbf{F}_2 acts at the point with position vector $(-3\mathbf{i} + 2\mathbf{k})\text{ m}$. The two forces are equivalent to a single force \mathbf{R} acting at the point with position vector $(\mathbf{i} + 2\mathbf{j} + \mathbf{k})\text{ m}$ together with a couple of moment \mathbf{G} .

Find,

(a) \mathbf{R} , (2)

(b) \mathbf{G} . (4)

A third force \mathbf{F}_3 is now added to the system. The force \mathbf{F}_3 acts at the point with position vector $(2\mathbf{i} - \mathbf{k})\text{ m}$ and the three forces \mathbf{F}_1 , \mathbf{F}_2 and \mathbf{F}_3 are equivalent to a couple.

(c) Find the magnitude of the couple. (6)



8. A pendulum consists of a uniform rod PQ , of mass $3m$ and length $2a$, which is rigidly fixed at its end Q to the centre of a uniform circular disc of mass m and radius a . The rod is perpendicular to the plane of the disc. The pendulum is free to rotate about a fixed smooth horizontal axis L which passes through the end P of the rod and is perpendicular to the rod.

(a) Show that the moment of inertia of the pendulum about L is $\frac{33}{4}ma^2$. (5)

The pendulum is released from rest in the position where PQ makes an angle α with the downward vertical. At time t , PQ makes an angle θ with the downward vertical.

(b) Show that the angular speed, $\dot{\theta}$, of the pendulum satisfies

$$\dot{\theta}^2 = \frac{40g(\cos \theta - \cos \alpha)}{33a} \quad (4)$$

(c) Hence, or otherwise, find the angular acceleration of the pendulum. (3)

Given that $\alpha = \frac{\pi}{20}$ and that PQ has length $\frac{8}{33}$ m,

(d) find, to 3 significant figures, an approximate value for the angular speed of the pendulum 0.2 s after it has been released from rest. (5)



